<u>Work and Energy</u>

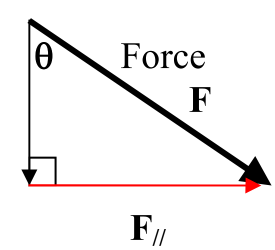
Work is done on an object that can exert a resisting force and is only accomplished if that object will move. In particular,

- we can describe *work done* <u>by</u> a specific object (where a force is <u>applied</u>) or <u>on</u> a specific object (where an opposing force must be overcome)
- we can also specify whether *work done* is due to one particular force or to the total <u>net force</u> on the object.
- *work done* is converted to other forms of energy.

Essentially, the amount of work accomplished can be determined ***two*** ways:

1) **Work = force x distance,** or **W = Fd** → units: **Joules (J)**

- Note that work is done <u>only</u> when a force acts *parallel* to the motion of an object, thereby affecting its motion. For any force that acts at an oblique angle to the direction of motion, only the parallel component of that force can be used to determine the work done.



→Use component $\mathbf{F}_{//}$ to find work: $\mathbf{W = F_{//}d}$

2) **Work = a change in energy,** or **W = ΔE** →the work-energy theorem

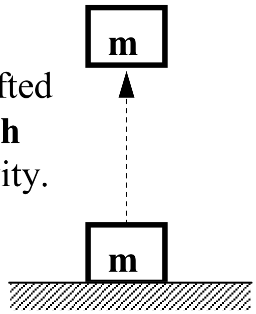
- this means that whenever one form of energy changes to another, work is done

Either method may be utilized to calculate work done, depending on the information given in the problem. What follows are examples of the *types* of work that can be done on an object.

A: Work Done Against Gravity

When an object is lifted upward, work is done on a mass against the resisting force of gravity. The energy used to do this is converted to *gravitational potential energy*, or $\mathbf{E_p}$. In fact, $\mathbf{E_p}$ increases as the mass is lifted higher and higher.

A mass is lifted a distance $\mathbf{\Delta h}$ against gravity.



→ start with $\mathbf{W = F_{App}d}$ where $\mathbf{F_{App}}$ is the applied force.

→ assuming the mass was lifted at a constant '**v**',

then $\mathbf{F_{Net} = 0}$ and $\mathbf{F_{App} = F_g}$

- therefore, the work done against gravity is $\mathbf{W = F_g d = mg\Delta h}$
- since the newly stored potential energy is $\mathbf{E_p = mgh}$, → $\mathbf{W = \Delta E_p}$

This tells us that work done against gravity = potential energy gained by the mass.

Example #1. A 6.0 kg mass is raised from 1.5 m above the ground to 6.5 m high.

a) What work is done?

b) What $\mathbf{E_p}$ does the mass now have?

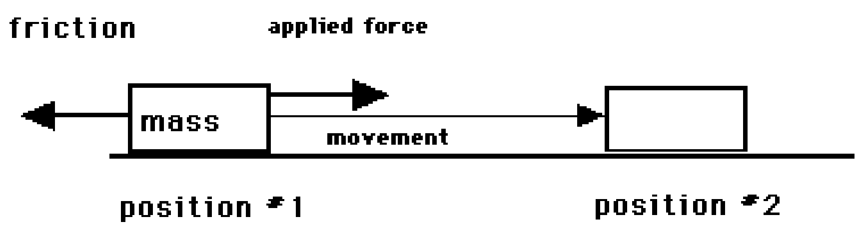
(see Work-Energy Ex 1 for answer)

Note: gravitational potential energy is a relative measurement which depends on what elevation is chosen to be $\mathbf{h = 0}$. Usually the '**0**' location is chosen as the lowest position that an object has the potential to fall.

B: Work done against the force of Friction

As long as an object moves along a horizontal surface with constant velocity, all the work is done against friction. If acceleration occurs, then work is being done against inertia as well (we will consider this later).

Consider the following diagram of an object moved from position #1 to position #2 at constant velocity.



As with the gravity example, start with $\mathbf{W = F_{App}d}$

- at constant velocity, there is no acceleration, so $\mathbf{F_{Net} = 0}$
- this means that $\mathbf{F_{App} = F_f}$ → $\mathbf{W = F_f d}$

The work done against friction is changed to heat energy and lost to the system.

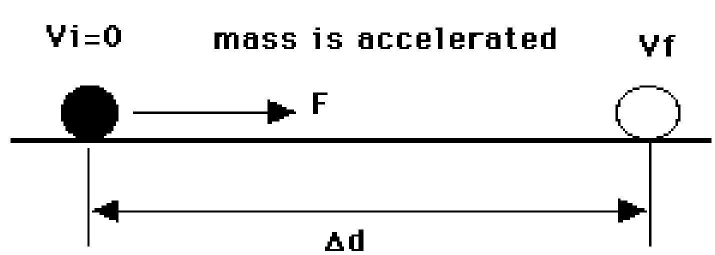
Example #2. A 150 kg object is pulled at constant velocity over a horizontal surface (μ = 0.12) for a distance of 7.0 m. How much heat energy was generated?

(see Work-Energy Ex 2 for answer)

C: Work done against inertia (also called *net* work done)

When a force acts to accelerate an object over a distance, that object is no longer "doing what it's already doing". In other words, work is done against *inertia*. This work is stored as the energy of speed, called *kinetic energy* or $\mathbf{E_k}$.

Consider the case of a ball upon which some unbalanced force acts. The ball is accelerated from $\mathbf{v_i=0}$ to some final speed $\mathbf{v_f}$ over a distance **d**.



Note that kinetic energy changes as the speed of the object changes. The work done against inertia to accelerate the mass a distance **d** can be determined *two* ways:

1) $\mathbf{W = F_{Net}d}$ 2) $\mathbf{W = \Delta E_k}$

To prove these two methods to find net work are the same:

- start with $\mathbf{F_{Net} = ma}$ → therefore $\mathbf{W = mad}$
- from kinematics, we also know that $\mathbf{a = \frac{v_f - v_i}{t}}$ and $\mathbf{d = (\frac{v_f + v_i}{2})t}$
- substituting into **W = mad,** we obtain

$$\mathbf{W = m(\frac{v_f - v_i}{t})(\frac{v_f + v_i}{2})t}$$ → **t** cancels, leaving

$$\mathbf{W = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$$

- and since $\mathbf{E_k = \frac{1}{2}mv^2}$, $\mathbf{W = \Delta E_k}$

In other words, <u>work is done against inertia to change kinetic energy</u>. If a force is exerted on a moving mass and its effect is to change the velocity of the mass, then work has been done against inertia. This is the *net* or <u>useful</u> work done.

Example #3: A 60.0 kg lab cart is moving at 5.00 m/s, and is accelerated to 12.0 m/s. How much work was done to cause this?

(see Work-Energy Ex 3 for answer)

Example #4: A force of 100 N is applied on a 50 kg cart that is moving with a speed of 6.0 m/s and has a force of friction of 20. N acting on it. At the end of 10. seconds, the cart is going 22 m/s.

a) **How much work was done against inertia?**
b) **How much work was done in total?**

(see Work-Energy Ex 4 for answer)

Total Work Done

The work in moving any object can be done against more than one resisting quantity. For example, when you accelerate a car up a steep hill, the car's engine is performing work against inertia, gravity and friction, all at the same time!

The total work done by the engine is

$\mathbf{W = \Delta E_k + \Delta E_p + F_f d}$ → calculate each of these quantities separately, then add them up.

However, keep in mind that when your engine applies a force to move the car up the steep hill, this total work can also be determined by

$\mathbf{W = F_{App} d}$ → where $\mathbf{F_{App}}$ is the force applied by the engine to move the car up the hill.

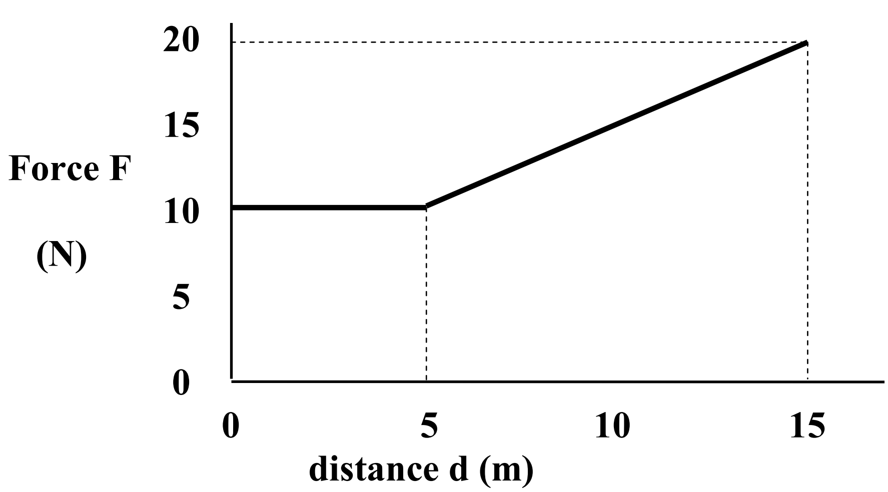
This second calculation can be used if you know what overall force is applied on an object, or can determine its value using vector analysis.

Work done when the force is not constant.

Because $\mathbf{W = Fd}$ and area $\mathbf{A = l \times w}$, work is the area under a force vs. distance graph. If an applied force is not constant, simply graph the varying force vs. distance and calculate the area; this gives total work done.

Example #5: A 5.0 kg cart is accelerated using a varying force. The force is a constant 10 N for 5 m, then increases at a constant rate up to 20 N for another 10 m.

a) What is the total work done on the cart?
b) If the cart was going 24 m/s when this began, what is its speed now?



(see Work-Energy Ex 5 for answer)